

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Sample Question Paper

Date – Morning/Afternoon

Version 2

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots + {}^n C_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Section A: Pure MathematicsAnswer **all** the questions1 Given that $f(x) = 6x^3 - 5x$, find

(a) $f'(x)$,

[2]

$$f'(x) = 18x^2 - 5$$

(b) $f''(2)$.

[2]

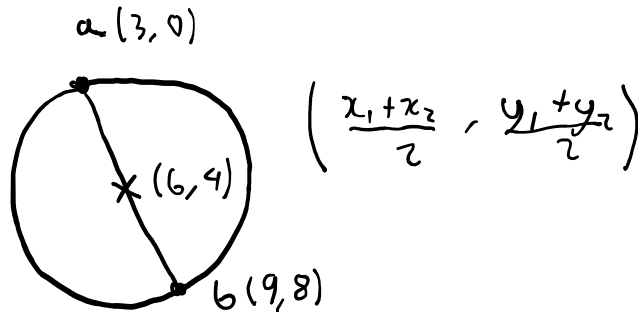
$$f''(x) = 36x$$

$$f''(2) = 36 \times 2 = 72$$

2 Points A and B have coordinates (3, 0) and (9, 8) respectively. The line AB is a diameter of a circle.

(a) Find the coordinates of the centre of the circle.

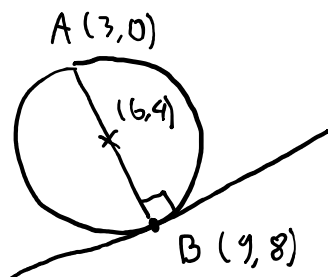
[2]



$$\left(\frac{3+9}{2}, \frac{0+8}{2}\right) = (6, 4)$$

(b) Find the equation of the tangent to the circle at the point B .

[3]



$$\text{Gradient of } AB = \frac{8-0}{9-3} = \frac{4}{3}$$

$$\text{Gradient of tangent} = -\frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{3}{4}(x - 9)$$

$$4(y - 8) = -3(x - 9)$$

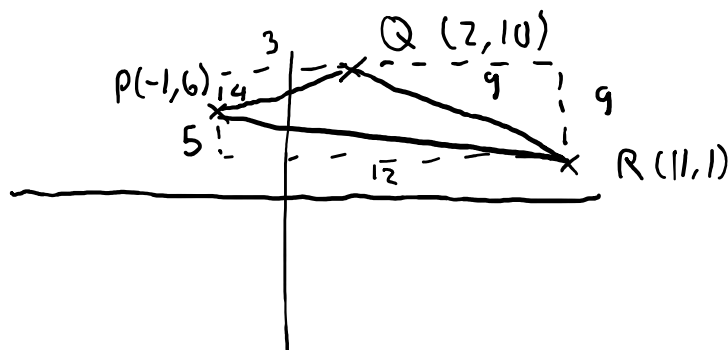
$$4y - 32 = -3x + 27$$

$$3x + 4y - 59 = 0$$

3 The points P , Q and R have coordinates $(-1, 6)$, $(2, 10)$ and $(11, 1)$ respectively.

Find the angle PRQ .

[4]



$$PR = \sqrt{5^2 + 12^2} = 13$$

$$PQ = 5$$

$$QR = \sqrt{9^2 + 9^2} = 9\sqrt{2}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$r^2 = p^2 + q^2 - 2pq \cos R$$

$$2pq \cos R = p^2 + q^2 - r^2$$

$$\cos R = \frac{p^2 + q^2 - r^2}{2pq}$$

$$\cos R = \frac{(9\sqrt{2})^2 + (13)^2 - (5)^2}{2 \times 9\sqrt{2} \times 13}$$

$$\cos R = \frac{306}{234\sqrt{2}}$$

$$R = 22.4^\circ \quad (3 \text{ s.f.})$$

- 4 The curve $y = 2x^3 + 3x^2 - kx + 4$ has a stationary point where $x = 2$.

(a) Determine the value of the constant k .

[5]

$$y = 2x^3 + 3x^2 - kx + 4$$

$$\frac{dy}{dx} = 6x^2 + 6x - k$$

$$\frac{dy}{dx} = 0$$

$$6x^2 + 6x - k = 0$$

$$6(2)^2 + 6(2) - k = 0$$

$$6(2)^2 + 6(2) = k = 36$$

- (b) Determine whether this stationary point is a maximum or a minimum point. [2]

$$\frac{d^2y}{dx^2} > 0 - \text{Min point} \quad \frac{d^2y}{dx^2} < 0 - \text{Max point}$$

$$\frac{dy}{dx} = 6x^2 + 6x - 36$$

$$\frac{d^2y}{dx^2} = 12x + 6$$

$$\frac{d^2y}{dx^2} = 12(2) + 6 = 30$$

\therefore The stationary point is a minimum point

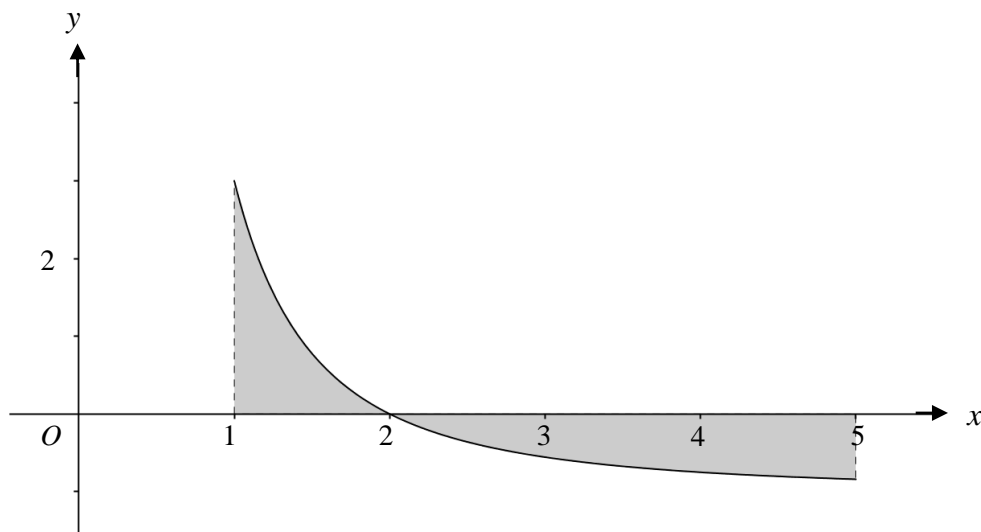
- 5 (a) Find $\int (x^3 - 6x) dx$. [3]

$$\begin{aligned} \int x^3 - 6x \, dx \\ = \left[\frac{x^4}{4} - 3x^2 \right] = \frac{1}{4} x^4 - 3x^2 + c \end{aligned}$$

- (b) (i) Find $\int \left(\frac{4}{x^2} - 1 \right) dx$. [3]

$$\begin{aligned} \int \frac{4}{x^2} - 1 \, dx &= \int 4x^{-2} - 1 \, dx \\ &= \left[-4x^{-1} - x \right] = -\frac{4}{x} - x + c \end{aligned}$$

- (ii) The diagram shows part of the curve $y = \frac{4}{x^2} - 1$.



The curve crosses the x -axis at $(2, 0)$.

The shaded region is bounded by the curve, the x -axis, and the lines $x = 1$ and $x = 5$.

Calculate the area of the shaded region.

[3]

$$\int_1^2 y \, dx - \int_2^5 y \, dx$$

$$\left[-\frac{2}{x} - x\right]_1^2 = \left[-\frac{2}{2} - 2\right] - \left[-\frac{2}{1} - 1\right] = 1$$

$$\left[-\frac{2}{x} - x\right]_2^5 = \left[-\frac{2}{5} - 5\right] - \left[-\frac{2}{2} - 2\right] = -\frac{9}{5}$$

$$1 - \left(-\frac{9}{5}\right) = 1 + \frac{9}{5} = \frac{14}{5} = 2.8$$

6 In this question you must show detailed reasoning.

The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 + 4x^2 + 7x - 5$.

- (a) Show that $(2x-1)$ is a factor of $f(x)$.

[2]

$$2x - 1 = 0 \quad f\left(\frac{1}{2}\right) = 0$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 5$$

$f\left(\frac{1}{2}\right) = 0 \quad \therefore 2x-1$ is a factor of $f(x)$ as the curve crosses the x -axis at this point.

(b) Hence solve the equation $4\sin^3\theta + 4\sin^2\theta + 7\sin\theta - 5 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

[7]

$$\text{Let } x = \sin\theta$$

$$\begin{array}{r}
 2x^2 + 3x + 5 \\
 \hline
 2x-1 \overline{) 4x^3 + 4x^2 + 7x - 5} \\
 \underline{- 4x^3 - 2x^2} \\
 0 6x^2 + 7x - 5 \\
 \underline{6x^2 - 3x} \\
 0 10x - 5 \\
 \underline{10x - 5} \\
 0 = 0
 \end{array}$$

$$F(x) = (2x-1)(2x^2+3x+5)$$

When $b^2 - 4ac < 0$, no real roots

$$a = 2 \quad b = 3 \quad c = 5$$

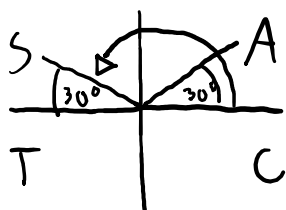
$$3^2 - 4(2)(5) = -31$$

$-31 < 0 \quad \therefore$ Has no real roots

$$\left(\frac{1}{2}, 0\right)$$

$$\begin{aligned}
 2x - 1 &= 0 \\
 2\sin\theta - 1 &= 0 \\
 \sin\theta &= \frac{1}{2}
 \end{aligned}$$

$$\theta = 30^\circ, 150^\circ$$



- 7 (a) Sketch the curve $y = 2x^2 - x - 3$. [3]

$$\begin{aligned} \text{When } y=0 \quad 2x^2 - x - 3 &= 0 \\ (2x-3)(x+1) &= 0 \\ x = \frac{3}{2} \quad x &= -1 \end{aligned}$$

- (b) Hence, or otherwise, solve $2x^2 - x - 3 < 0$. [2]

$$-1 < x < \frac{3}{2}$$

- (c) Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of k . [3]

$$2x^2 - x - 3 = k$$

$$2x^2 - x - 3 - k = 0$$

$$a=2 \quad b=-1 \quad c=-3-k$$

$$b^2 - 4ac < 0 \quad \text{No real roots}$$

$$(-1)^2 - 4(2)(-3-k) < 0$$

$$1 - 8(-3-k) < 0$$

$$1 + 24 + 8k < 0$$

$$25 + 8k < 0$$

$$8k < -25$$

$$k < \frac{-25}{8}$$

Section B: StatisticsAnswer **all** the questions

8 A club secretary wishes to survey a sample of members of his club. He uses all members present at a particular meeting as his sample.

(a) Explain why this sample is likely to be biased.

[1]

Members who attend the club may be of a particular type

Later the secretary decides to choose a random sample of members.

The club has 253 members and the secretary numbers the members from 1 to 253. He then generates random 3-digit numbers on his calculator. The first six random numbers generated are 156, 965, 248, 156, 073 and 181. The secretary uses each number, where possible, as the number of a member in the sample.

(b) Find possible numbers for the first four members in the sample.

[2]

76, 156, 181, 248

9 The probability distribution of a random variable X is given in the table.

x	1	2	3
$P(X = x)$	0.6	0.3	0.1

Two values of X are chosen at random.

Find the probability that the second value is greater than the first.

[3]

(1 and 2) or (1 and 3) or (2 and 3)

$(0.6 \times 0.3) + (0.6 \times 0.1) + (0.3 \times 0.1)$

$= 0.27$

- 10 (a) Write down and simplify the first four terms in the expansion of $(x+y)^7$.
Give your answer in ascending powers of x . [2]

$$\begin{aligned}(x+y)^7 &= y^7 + {}^7C_1 xy^6 + {}^7C_2 x^2 y^5 + {}^7C_3 x^3 y^4 \\ &= y^7 + 7xy^6 + 21x^2y^5 + 35x^3y^4\end{aligned}$$

- (b) Given that the terms in x^2y^5 and x^3y^4 in this expansion are equal, find the value of $\frac{x}{y}$. [2]

$$21x^2y^5 = 35x^3y^4$$

$$21 = \frac{35x}{y}$$

$$\frac{21}{35} = \frac{x}{y} = \underline{\underline{\frac{3}{5}}}$$

- (c) A hospital consultant has seven appointments every day.
The number of these appointments which start late on a randomly chosen day is denoted by L .
The variable L is modelled by the distribution $B(7, \frac{3}{8})$.

Show that, in this model, the hospital consultant is equally likely to have two appointments start late or three appointments start late. [3]

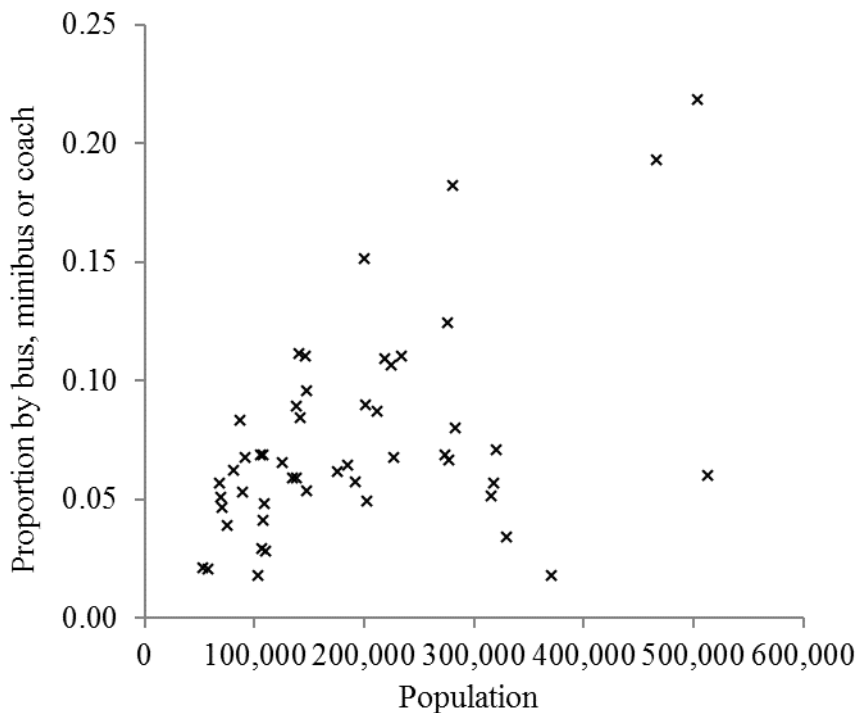
$$L \sim B(7, \frac{3}{8})$$

$$P(L=2) = 0.2816$$

$$P(L=3) = 0.2816$$

As the probabilities are the same
the hospital consultant is equally likely
to have 2 or 3 appointments start late.

- 11** The scatter diagram below shows data taken from the 2011 UK census for each of the Local Authorities in the North East and North West regions. The scatter diagram shows the total population of the Local Authority and the proportion of its workforce that travel to work by bus, minibus or coach.



- (a) Samuel suggests that, with a few exceptions, the data points in the diagram show that Local Authorities with larger populations generally have higher proportions of workers travelling by bus, minibus or coach.

On the diagram in the Printed Answer Booklet draw a ring around each of the data points that Samuel might regard as an exception. [1]

- (b) Jasper suggests that it is possible to separate these Local Authorities into more than one group with different relationships between population and proportion travelling to work by bus, minibus or coach.

Discuss Jasper's suggestion, referring to the data and to how differences between the Local Authorities could explain the patterns seen in the diagram. [3]

Circle 3 points in the bottom right.

E06 - Unitary authority in England

= - Non-metropolitan district in England

E08 - Metropolitan borough in England

E09 - London Boroughs

W06 - Unitary authority in Wales

Jasper may suggest to separate the local authorities between metropolitan districts, non-metropolitan districts and unitary authorities. As seen in the diagram it suggests that the metropolitan districts which have a higher population of people have good transport services. Therefore, a higher proportion of people travel by bus. The diagram also shows data points where there is a high population but a smaller proportion of people travel by bus. These points could be made up of large unitary authorities with poor transport services.

Data points with a small population and a small proportion of people travelling by bus are made up of small unitary authorities and non-metropolitan districts which are difficult to tell apart.

- 12 It is known that under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year. In a trial of a new treatment, 450 patients with this disease were selected and the number, X , that experienced side effects within one year was noted. It was found that 51 of the 450 patients experienced side effects within one year.

- (a) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment.

[7]

Let p be the proportion of patients who experience side effects.

$$H_0: p = 0.097$$

$$H_1: p > 0.097$$

$$X \sim B(450, 0.097)$$

$$P(X \geq 51) = 1 - P(X \leq 50)$$

$$= 1 - 0.8616986812$$

$$= 0.138 \text{ (3 s.f.)}$$

\therefore as $0.138 > 0.10$ we fail to reject H_0 . Therefore, there is insufficient evidence to suggest that the proportion of patients under the new treatment had greater side effects compared to the standard treatment.

- (b) It was later discovered that all 450 patients selected for the trial were treated in the same hospital.

Comment on the validity of the model used in part (a).

[1]

b) The binomial distribution may not be valid as all 450 patients could be treated together. Therefore, they are not independent.

- 13 Clara used some data from the 2011 UK census to summarise information on carbon emissions due to travel to work, in two Local Authorities.

Her results are shown below.

		Method of travel to work	Individual motorised transport	Shared motorised transport	Public transport	No motorised transport	
		Carbon emissions category	High	Medium	Low	None	Total
Local Authority A	Number of workers		174 374	42 112	61 483	76 024	353 993
	Percentage of workers		49.3	11.9	17.4	21.5	100
Local Authority B	Number of workers		39 433	9944	4614	16 232	70 223
	Percentage of workers		56.2	14.2	6.6	23.1	100

- (a) Clara calculated the values for the column headed “shared motorised transport” by doubling the value in the “passenger in a car or van” column of the original data set.

Explain what assumption she has made and what other adjustment would need to be made to the data to take account of this.

[2]

She has assumed that any car or van is made up of exactly 2 people.

The original data can be adjusted by subtracting the number of passengers in a car or van from the number of people in a car to find the number of people driving alone.

(b) Clara suggests that the average carbon emissions per worker due to travelling to work is larger in region B than in region A.

(i) Use data from the table to support Clara's suggestion.

[1]

The proportion of people using motorised transport in region B is greater than region A.

(ii) Use data from the table to argue against Clara's suggestion.

[1]

The proportion of people using no motorised transport in region B is greater than region A.

END OF QUESTION PAPER